St. Agnes Academy

Honors Algebra II
Summer Math Packet

http://www.montgomeryschoolsmd.org/schools/kingsviewms/documents/Math/Algebra%202_Hon%20Algebra%202%20Summer%20Packet%202010.pdf

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Directions: Complete all problems WITHOUT using a calculator. Show all work. Turn in completed packeted with ALL work the first day of class.

Order of Operations

Hints/Guide:

The rules for multiplying integers are:

positive x positive = positive   negative x negative = positive
positive x negative = negative   negative x positive = negative

The rules for dividing integers are the same as for multiplying integers.

REMEMBER: Order of Operations (PEMDAS)

P – parentheses
E – exponents
M/D – multiply/divide which comes first
A/S – add/subtract which comes first

Exercises: Solve the following problems. Show all work.

1. $\frac{100-15}{9+8}$

2. $3+4[13-2(6-3)]$

3. $32 \div (-7+5)^3$

4. $14+6 \bullet 2-8 \div 4$

Use grouping symbols to make the equation true.

5. 6 + 8 ÷ 4 • 2 = 7

Solving Equations

Hints/Guide:

Equation Solving Procedure:

1. Multiply on both sides to clear the equation of fractions or decimals.
2. Distribute.
3. Collect like terms on each side, if necessary.
4. Get all terms with variables on one side and all constant terms on the other side.
5. Multiply or divide to solve for the variable.
6. Check all possible solutions in the original equation.

Example: $5(x-2)+7=3(x+1)-2$ Distribute.

$5x-10+7=3x+3-2$ Combine like terms. Simplify.

$5x-3=3x+1$ Move all terms with variables to one side.

$2x=4$ Divide to isolate the variable.

$x=2$

Exercises: Solve each equation. Show all work.

1. $3(r-6)=-21$

2. $5(t+3)+9=-6$

3. $2(x+4)-20=-3(x-6)$

4. $a+(a-3)=a+2-(a+1)$

Exponents

Hints/Guide:

Rules of Exponents

$a^0 = 1$ $a^1 = a$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$

Product Rule: $a^m a^n = a^{m+n}$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Power Rule: $(a^m)^n = a^{mn}$

Quotient to a Power: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Product to a Power: $(ab)^n = a^n b^n$

Exercises: Simplify using the Rules of Exponents.

1. $6^{-2} \bullet 6^{-3}$
2. $x^6 \bullet x^2 \bullet x$
3. $(4a)^3 \bullet (4a)^8$
4. $\frac{3^5}{3^2}$
5. $\frac{x^3}{x^8}$
6. $\frac{(2x)^5}{(2x)^5}$
7. $(x^3)^2$
8. $(-3y^2)^3$
9. $(2a^3b)^4$
10. $(3x^2)^3(-2x^5)^3$
11. $(2x^3y^{-2})^3$
12. $(2x)^2(-3x)^4$

Express using a positive exponent.

13. 5^{-3}
14. $\frac{1}{y^{-8}}$

Addition, Subtraction and Multiplication of Polynomials

Hints/Guide:

- Only like terms can be added or subtracted.
- Like terms have the same variables with the same exponents.
- Only the coefficients (numbers) are added or subtracted.
- A subtraction sign in front of the parentheses changes each term in the parentheses to the opposite.
- Multiply the coefficients and use the rules of exponents for the variables.
- Remember: FOIL F – first O – outers I – inners L – last **OR** Box Method

Examples:

1) Add the polynomials.

$$(3x^2 - 2x + 2) + (5x^3 - 2x^2 + 3x - 4)$$
$$= 5x^3 + 3x^2 - 2x^2 - 2x + 3x + 2 - 4$$
$$= 5x^3 + x^2 + x - 2$$

2) Subtract the polynomials.

$$(9x^5 + x^3 - 2x^2 + 4) - (2x^5 + x^4 - 4x^3 - 3x^2)$$
$$= (9x^5 + x^3 - 2x^2 + 4) - 2x^5 - x^4 + 4x^3 + 3x^2$$
$$= 7x^5 - x^4 + 5x^3 + x^2 + 4$$

3) Multiply the polynomials.

$$(x^2 + 4)(x^2 + 2x - 3)$$
$$= x^2(x^2 + 2x - 3) + 4(x^2 + 2x - 3)$$
$$= x^4 + 2x^3 - 3x^2 + 4x^2 + 8x - 12$$
$$= x^4 + 2x^3 + x^2 + 8x - 12$$

Exercises: Add, subtract, or multiply the polynomials. Show all work.

1. $(3x + 2) + (-4x + 3)$
2. $(-6x + 2) + (x^2 + x - 3)$
3. $(6x + 1) - (-7x + 2)$
4. $(3x^2 - 5x + 4) - (x^2 + 8x - 9)$
5. $-3x(x - 1)$
6. $-4x(2x^3 - 6x^2 - 5x + 1)$
7. $(x + 5)(x - 2)$
8. $(x - 5)(2x - 5)$
9. $(x - 1)(x^2 + x + 1)$
10. $(x + 5)^2$

Quadratic Formula

Hints/Guide:

- **Assume that the radical extends over the whole expression $b^2 - 4ac$.**
- Equation must be in the form $ax^2 + bx + c$ = 0 (standard form) to begin.
- Try to factor first.
- If you cannot find factors, then use the quadratic formula.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

Solve $x^2 = 4x + 7$

Write the equation in standard form: $x^2 - 4x - 7 = 0$

Identify a, b, and c for the formula: a = 1, b = -4, c = -7

Substitute into the formula: $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$

Simplify: $x = \frac{4 \pm \sqrt{16 + 28}}{2}$

Separate into two solutions: $x = \frac{4 + \sqrt{44}}{2}$ and $x = \frac{4 - \sqrt{44}}{2}$

Solutions: $x = 5.32$ and $x = -1.32$

Exercises: Solve using the quadratic formula. Show all work.

1. $x^2 - 4x = 21$

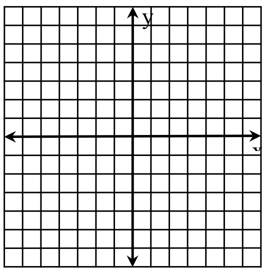
2. $x^2 = 6x - 9$

3. $3y^2 - 7y + 4 = 0$

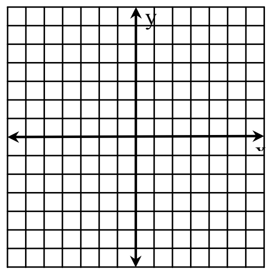
Graphing Functions

Use slope (m) and y-intercept (b) to graph the following linear equations y = mx + b.

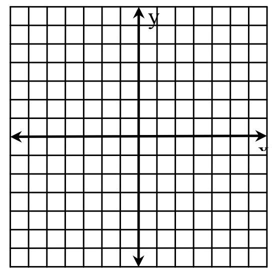
1. $y = x + 1$



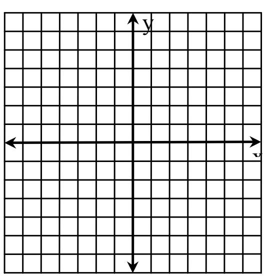
2 $y = 2x - 3$



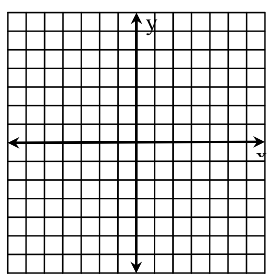
3. $y = 5x - 2.5$



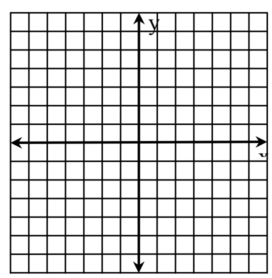
4. $y = -3x + 1$



5. $y = \frac{1}{4}x - 1$

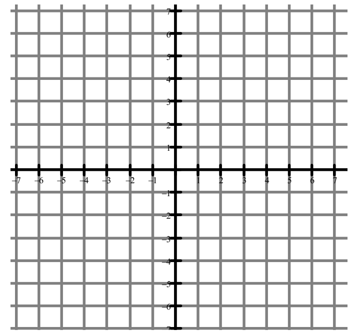


6. $y = \frac{2}{3}x + 2$



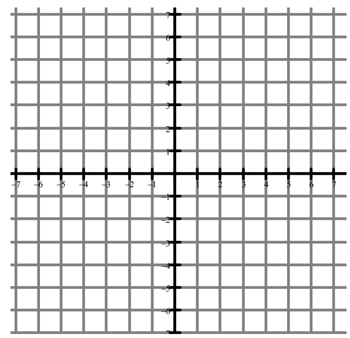
7. y = $|x|$

X	Y
-2	
-1	
0	
1	
2	



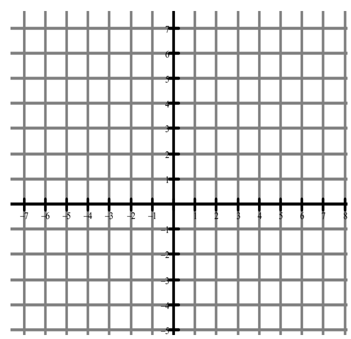
8. y = x^2

X	Y
-2	
-1	
0	
1	
2	



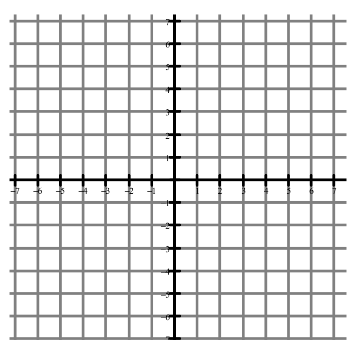
9. y = $\sqrt{x}$

X	Y
-1	
0	
1	
4	
9	



10. y = 2^x

X	Y
-1	
0	
1	
2	
3	



Radicals

Hints/Guide:

Roots or radicals are the opposite operation of applying exponents; you can "undo" a power with a radical, and a radical can "undo" a power. For instance, if you square 2, you get 4, and if you "take the square root of 4", you get 2. To simplify a square root, you "take out" anything that is a "perfect square"; that is, you take out front anything that has two copies of the same factor.

Ex.- $\sqrt{25x^2}$ = $\sqrt{5 \cdot 5 \cdot x \cdot x}$ = $5x$

To simplifying multiplied radicals,we use the fact that the product of two radicals is the same as the radical of the product, and vice versa. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Ex.- $\sqrt{24}\sqrt{6} = \sqrt{144} = 12$

Just as with regular numbers, square roots can be added together. But you might not be able to simplify the addition all the way down to one number. Just as "you can't add apples and oranges", so also you cannot combine "unlike" radicals. To add radical terms together, they have to have the same radical part.

Ex.- $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

Exercises: Simplify the radicals.

1. $\sqrt{9}$
2. $\sqrt{144}$
3. $\sqrt{196}$
4. $\sqrt{49}$
5. $\sqrt{25x^2}$
6. $\sqrt{27y^3}$
7. $\sqrt{64x^5}$
8. $\sqrt{12x^7y^6z}$

Multiply the radicals.

9. $\sqrt{2}\sqrt{8}$
10. $\sqrt{3}\sqrt{6}$
11. $\sqrt{6}\sqrt{15}\sqrt{10}$
12. $\sqrt{4x}\sqrt{5x^3}$
13. $\sqrt{5xy^2}\sqrt{15x^2y}$
14. $\sqrt{4x^4}\sqrt{16x^3}$

Add or subtract the radicals.

15. $\sqrt{3} + 3\sqrt{3}$
16. $\sqrt{9} + \sqrt{25}$
17. $\sqrt{5} + 2\sqrt{3} + 3\sqrt{5}$
18. $3\sqrt{4} - 2\sqrt{4}$
19. $\sqrt{9} - \sqrt{4}$
20. $\sqrt{8} + 5\sqrt{2}$

Factoring Polynomials

Hints/Guide:

- Always look for the greatest common factor first.
- Don't forget to include the variable in the common factor.
- Factor into two parentheses, if possible.
- Check your answer by multiplying.

Examples:

Factor $15x^5 + 12x^4 + 27x^3 - 3x^2$

Question: What factor is common to the coefficients of 15, 12, 27, and 3?

Answer: 3

Question: What exponent is common to variables of x^5, x^4, x^3, and x^2?

Answer: x^2

= $3x^2(5x^3 - 4x^2 + 9x - 1)$

Factor $t^2 + 5t - 24$ Think: What multiplies to -24 and adds to +5?

= $(t-3)(t+8)$

Pairs of Factors	Sums of Factors
-1, 24	23
-2, 12	10
-3, 8	5
-4, 6	2

Exercises: Find the GCF from the lists of factors for each pair of numbers.

1. 12: 1, 2, 3, 4, 6, 12
   8: 1, 2, 3, 4, 6, 9, 18

2. 36:1, 2, 3, 4, 6, 9, 12, 18, 36
   54:1, 2, 3, 6, 9, 18, 27, 54

3. 8: 1, 2, 4 ,8
   12: 1, 2, 3, 4, 6, 12

Factor the polynomials. Show all work.

1. $21x + 35$
2. $x^2 - 4x$
3. $10x^2 - 5x$
4. $x^2 + 5x + 6$
5. $y^2 - 81$
6. $x^2 - 8x + 15$
7. $x^2 + 2x - 15$
8. $2x^2 + 8x + 6$
9. $2x^2 - 7x - 4$

1. Line k passes through the point (8, -3) and is parallel to the line y = 3x – 4. Write an equation for line k.

2. Line *m* is perpendicular to y = 4x – 1and passes through the origin. What is the equation of line *m?*

3. Use A = $\begin{bmatrix} 12 & 7 \\ -1 & -3 \end{bmatrix}$ and B = $\begin{bmatrix} -5 & 6 \\ 14 & 0 \end{bmatrix}$ to perform the indicated operations.
   a. A + B

   b. 2B – A

   c. -4A

4. Simplify the expressions.
   a. $(x^3 + 3x^2 - 2) + (5x^3 + x + 8) - (9x^3 - x^2 + 4)$

   b. $(4x - 3y)(x + 5y)$

   c. $(3x^2 + x + 1)(2x - 1)$

   d. $\dfrac{16x^4 - 12x^5y^3}{2x^3y^2}$

   e. $(5x - 2)^2$

   f. $2(x^3 - 5x^2 + 6x) - (x^2 + 3x)$

5. Factor completely.

   a. $9x^2y^3 - 3x^3y^2 - 15xy$

   b. $2x^2y - 4xy - 30y$

   c. $x^2 - x - 30$

   d. $4x^2 - 81$

   e. $x^3 + 4x^2 + 3x$

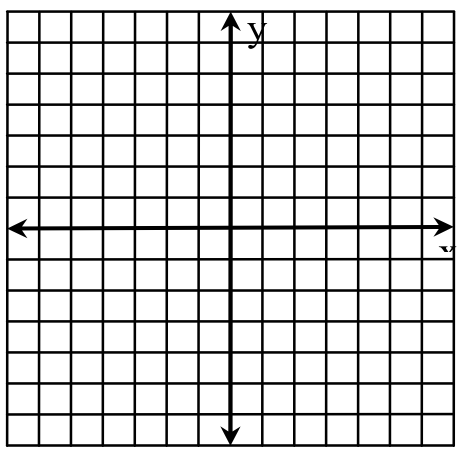
   f. $2x^2 - 5x - 3$

6. A car salesman's weekly salary is base amount plus an additional amount for each car sold. The table below shows a person's weekly salary earned for the last three weeks.

Cars Sold (C)	Weekly Salary (S)
4	\$500
9	\$1000
12	\$1300

What is the person's weekly salary when 13 cars are sold? Justify your answer.

7. Sketch a graph of $f(x) = x^2 - x - 2$. Then complete the characteristics below.



Domain:

Range:

Axis of Symmetry:

Increasing Interval:

Decreasing Interval

x-intercepts:

y-intercept:

Minimum Value:

Maximum Value:

Vertex:

Continuous?

8. What can you say about the x-coordinates of two distinct points on a vertical line?

9. Simplify each of the following using exact answers – no decimals. (Leave your answers in radical form.)

a. $\sqrt{32}$ b. $\sqrt{\frac{9}{4}}$ c. $\sqrt{\frac{3}{2}}$ d. $\sqrt{8} + \sqrt{18} - \sqrt{32}$

e. $\sqrt{21} \bullet \sqrt{14}$